Chiral perturbation theory for two-color QCD at non-zero baryon density

Tomas Brauner

brauner@ujf.cas.cz

Nuclear Physics Institute, Rez
Czech Republic

Outline

- what is two-color QCD and why is it worth studying
- basic properties of 2C QCD
  - global symmetry
  - symmetry breaking patterns
- 2C QCD at finite density
- chiral perturbation theory
  - low-energy degrees of freedom
  - effective Lagrangian
- two-flavor case
- references

QCD at non-zero density

- ▷ want to study the phase diagram of QCD
  - low temperature and density — confinement
  - high temperature — quark-gluon plasma
  - high density — color superconductivity
- ▷ scale set by the RG invariant Λ_{QCD}
- ▷ analytical *ab initio* calculations not available in the strongly coupled regime
- ▷ lattice simulations confirm the deconfinement phase transition on the temperature axis
- ▷ very little known about the structure of the phase diagram near the density axis
- ▷ current lattice methods fail at high density because the determinant of the Euclidean Dirac operator

$$\mathcal{D} = \gamma_\nu D_\nu + m - \mu\gamma_0$$

  is complex
- ▷ investigate similar theories possessing a positive fermion determinant and hope that the mechanism of confinement and chiral symmetry breaking is the same

Chiral limit

- an $SU(2)$ gauge theory with quarks in doublets
- the Euclidean Lagrangian in the chiral limit

$$\mathcal{L} = \bar{\psi}\gamma_\nu D_\nu \psi$$

has an apparent $U(N_f)_L \times U(N_f)_R$ symmetry

- the fundamental representation of the gauge group is pseudoreal,

$$\tau_k^* = -\tau_2 \tau_k \tau_2$$

- instead of the Dirac bispinor $\psi = (\psi_L \ \psi_R)^T$, use

$$\Psi \equiv \begin{pmatrix} \psi_L \\ \sigma_2 \tau_2 \psi_R^* \end{pmatrix}$$

- the Lagrangian becomes

$$\mathcal{L} = \mathrm{i}\Psi^\dagger \sigma_\nu D_\nu \Psi,$$

being now manifestly $U(2N_f)$ invariant

- diagonal $U(1)$ broken by the axial anomaly $\Rightarrow$ 2C QCD as a quantum theory has an extended $SU(2N_f)$ symmetry

Extended $SU(2N_f)$ symmetry

▷ naturally incorporates baryon number $U(1)_B$ generated by

$$B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

▷ mixes quarks and antiquarks i.e., mesons and baryons

▷ at zero density broken by the standard chiral condensate,

$$\langle \bar{\psi}\psi \rangle = -\left\langle \frac{1}{2} \Psi^T \sigma_2 \tau_2 \underbrace{\left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix} \right)}_{M} \Psi \right\rangle + c.c.$$

▷ symmetry breaking pattern $SU(2N_f) \to Sp(2N_f)$, resulting in $2N_f^2 - N_f - 1$ Goldstone bosons, in contrast to the $N_f^2 - 1$ naively expected

▷ the GBs span an irreducible multiplet of the unbroken $Sp(2N_f)$, which consists of

- $N_f^2 - 1$ "ordinary" mesons — pions
- $N_f^2 - N_f$ baryons — diquarks

▷ the existence of GBs carrying baryon number is essential for the investigation of the phase diagram within the low-energy effective theory

Non-zero (equal) quark masses

- ▷ introduce a mass term into the Lagrangian,

$$m\bar{\psi}\psi = -\frac{1}{2}m\Psi^T\sigma_2\tau_2 M\Psi + h.c.$$

- ▷ explicit breaking $SU(2N_f) \to Sp(2N_f)$
- ▷ all GBs become "pseudo" (receive equal masses)

Non-zero density (chemical potential)

- ▷ add a term to the Lagrangian,

$$-\mu\psi^\dagger\psi = -\mu\Psi^\dagger B\Psi$$

- ▷ explicit breaking of the global symmetry

$$SU(2N_f) \to SU(N_f)_L \times SU(N_f)_R \times U(1)_B,$$

mean quark number fixed by $\mu \Rightarrow$ quark-antiquark mixing no more allowed

- ▷ promote the $SU(2N_f)$ symmetry to a local one,

$$\mathcal{L} = \mathrm{i}\Psi^\dagger\sigma_\nu(D_\nu - \mu B_\nu)\Psi,$$

$$\Psi \to U\Psi, \quad B_\nu \to UB_\nu U^\dagger - \frac{1}{\mu}U\partial_\nu U^\dagger,$$

and eventually set $B_\nu = (B, \vec{0})$

Chiral perturbation theory

- ▷ effective theory for the low energy degrees of freedom — the Goldstone bosons
- ▷ power expansion in the momentum/energy of the GBs
- ▷ the GBs are long-wavelength fluctuations of the symmetry-breaking order parameter — the vacuum condensate

$$\frac{1}{2}\langle \Psi_i^T \sigma_2 \tau_2 \Psi_j \rangle \sim \langle \Sigma_{ij} \rangle$$

- ▷ parameterize the GBs by a unitary antisymmetric matrix Σ that transforms under $SU(2N_f)$ as

$$\Sigma \to U \Sigma U^T$$

- ▷ to lowest order $\mathcal{O}(p^2)$, the most general effective Lagrangian consistent with the underlying symmetries is

$$\mathcal{L}_{\text{eff}}^{(2)} = \frac{F^2}{2} \operatorname{Tr} \partial_\nu \Sigma \partial_\nu \Sigma^\dagger$$

- ▷ find the vacuum condensate $\overline{\Sigma} \equiv \langle \Sigma \rangle$, parameterize

$$\Sigma = U(\pi) \overline{\Sigma} U^T(\pi)$$

and expand in terms of the GB fields π

Incorporation of quark masses and chemical potential

▷ standard treatment of the quark mass term

- make it $SU(2N_f)$ invariant by the formal transformation $M \to U^* M U^\dagger$
- treat quark masses as $\mathcal{O}(p^2)$ $\Rightarrow$ construct the invariant term for $\mathcal{L}_{\text{eff}}^{(2)}$ as $\mathrm{Tr}(M\Sigma)$
- vacuum alignment in the presence of the mass term: $\overline{\Sigma} = M^\dagger$

▷ dependence on μ uniquely fixed by the gauge invariance of the microscopic 2C QCD Lagrangian — it can only enter as a part of the covariant derivative

$$\nabla_\nu \Sigma \equiv \partial_\nu \Sigma - \mu(B_\nu \Sigma + \Sigma B_\nu^T)$$

▷ the full $\mathcal{O}(p^2)$ Lagrangian reads

$$\mathcal{L}_{\text{eff}}^{(2)} = \frac{F^2}{2}\left[\mathrm{Tr}\, \nabla_\nu \Sigma \nabla_\nu \Sigma^\dagger - 2m_\pi^2 \,\mathrm{Re}\,\mathrm{Tr}(M\Sigma)\right]$$

▷ two free parameters in $\mathcal{L}_{\text{eff}}$ — F and m_π, no additional parameter associated with μ

▷ $\overline{\Sigma}$ determined by minimizing the static Lagrangian

$$\mathcal{L}_{\text{stat}}^{(2)} = -F^2\mu^2\,\mathrm{Tr}(\Sigma B \Sigma^\dagger B + BB) - F^2 m_\pi^2\,\mathrm{Re}\,\mathrm{Tr}(M\Sigma)$$

Results at the leading order

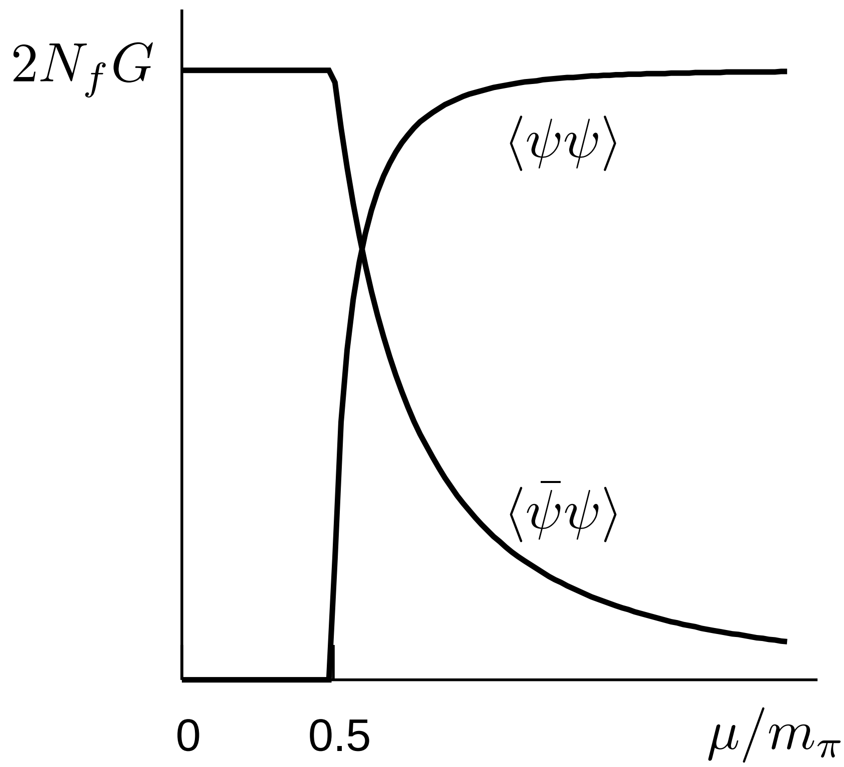
Structure of the ground state

$\mu < m_\pi/2$ — standard "vacuum" chiral condensate $\langle\bar{\psi}\psi\rangle$

$\mu > m_\pi/2$ — the condensate starts rotating into the diquark condensate $\langle\psi\psi\rangle$

$$\langle\bar{\psi}\psi\rangle = 2N_f G\cos\alpha \qquad \cos\alpha = \left(\frac{m_\pi}{2\mu}\right)^2$$

$$\langle\psi\psi\rangle = 2N_f G\sin\alpha$$

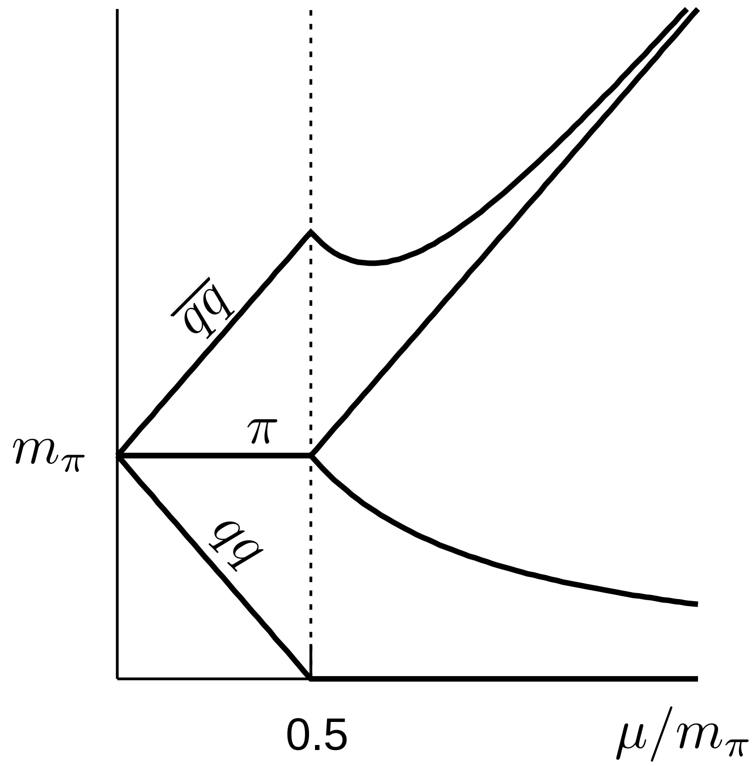


Baryon number $U(1)_B$ broken for $\mu > m_\pi/2$ by the diquark condensate. Non-zero baryon density

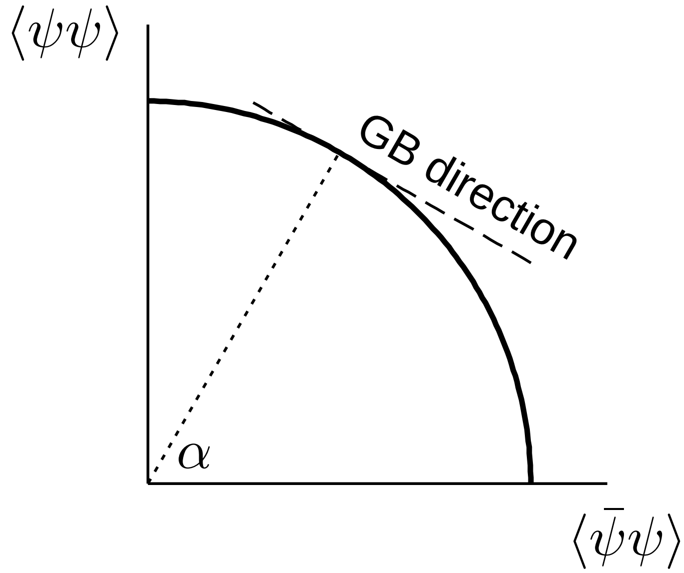
$$n_B = 8N_f F^2\mu\sin^2\alpha.$$

Excitations

Pions and diquarks, their effective mass depends on μ. For $\mu < m_\pi/2$ all are pseudoGBs. For $\mu > m_\pi/2$ there are massless GBs from the broken exact $SU(N_f)_V \times U(1)_B$.



The nature of the GBs changes as μ increases.



Two-flavor case

- note the algebra isomorphisms $SU(4) \simeq SO(6)$ and $Sp(4) \simeq SO(5)$
  - construct an $SO(6)/SO(5)$ effective theory
  - parameterize the GBs by a unit 6-vector $\vec{n}$
- makes the ground state structure easier to understand (minimize the static Lagrangian on a unit sphere)
- makes the nature of the GBs more transparent
- connection with the $SU(4)/Sp(4)$ formalism provided by

$$\Sigma = \vec{n} \cdot \vec{\Sigma},$$

  where $\vec{\Sigma}$ are 6 independent antisymmetric 4×4 matrices
- at zero density there are five GBs — three mesons (pions) and two baryons (diquarks), which form a real pentaplet of $SO(5)$
- depending on μ, quark masses and other (i.e. isospin) sources, the vacuum condensate rotates on the unit sphere
- the 6 basic condensates include scalar $\langle\bar{\psi}\psi\rangle$, pseudoscalar isospin triplet, and a complex $U(1)_B$-charged (diquark) scalar

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